We note also that the foregoing procedure, with particular reference to (4), (7), and (9) applies to nonuniform multiconductor lines as well; while the numerical determination of a suitable fundamental matrix is not straightforward [7], the technique is systematic.

III. APPLICATIONS

With the view of illustrating the procedure with the aid of examples which may be readily verified, we consider first coupled microstrip lines for which scattering parameters have been presented by Napoli [8] and Levy [9] using different procedures. The problem can be reduced to finding voltages and currents at $x = 0$ and $x = L$ of a single transmission line of length L, having either an odd mode characteristic impedance Z_{0o} or an even mode impedance Z_{0e}, a voltage source at one end and terminated in impedances Z_o at both ends. It can be very easily solved applying (10)–(15) when it is noted that each matrix reduces to a single number. Thus we find with the aid of (4a) and (9), letting $I_S = 0$ and assuming for generality that the terminations are Z_a and Z_b, respectively, rather than identical (while the characteristic impedance is Z_0), that

$$V(x) = \frac{E_S(0)Z_0 \{\exp(-\gamma x) + \rho_B \exp[\gamma(x - 2L)]\}}{(Z_a + Z_0)[1 - \rho_A\rho_B \exp(-2\gamma L)]} \quad (16)$$

$$I(x) = \frac{E_S(0) \{\exp(-\gamma x) - \rho_B \exp[\gamma(x - 2L)]\}}{(Z_a + Z_0)[1 - \rho_A\rho_B \exp(-2\gamma L)]} \quad (17)$$

where γ represents one of the two eigenvalues of the problem obtained by [10] solving $\det[\gamma^2 U - ZY] = 0$ and $\rho_A = (Z_a - Z_0)/(Z_a + Z_0)$, while $\rho_B = (Z_b - Z_0)/(Z_b + Z_0)$.

As a second example arising in connection with the Chipman's method [11],[12] of impedance measurement we consider a single transmission line of length L terminated in Z_a at $x = 0$ and Z_b at $x = L$; voltage is induced from a loop assumed to be located at a point ξ and it is desired to find the current at a point $x > \xi$.

Applying (10)–(15) and noting that each matrix reduces to a single number we find easily that

$$I(x) = \frac{E_S(\xi)}{2Z_0[1 - \rho_A\rho_B \exp(-2\Gamma L)]} \times \{\exp[\Gamma(\xi - x)] - \rho_A \cdot\exp[-\Gamma(x + \xi)] - \rho_B \exp[\Gamma(x + \xi - 2L)] + \rho_A\rho_B \cdot\exp[\Gamma(x - \xi - 2L)]\} \quad (18)$$

where ρ_A and ρ_B have the same significance as before; this result has been derived by Jackson [11] with some changes of notation.

Finally, it may be observed that when W^a and W^b are suitably chosen, letting $E_S = 0$ and $I_S = 0$ in turn, yields the open-circuit and short-circuit matrices of the system, respectively.

Thus letting $E_S = 0$ and noting that in this case

$$V(x) = \int_0^L G_{12}(x,\xi) I(\xi)\, d\xi \quad (19)$$

if $I(\xi)$ represents a source of unit magnitude (and is interpreted as n unit vectors times $\delta\xi$) located at $\xi = 0$ and $\xi = L$, respectively, then $V(x)$ is numerically equal to $G_{12}(x,\xi)$ and all open-circuit parameters are arrived at by a suitable choice of x and ξ. For example, for a single transmission line it is easily verified that

$$G_{12}(x,\xi) = \frac{Z^{1/2}Y^{-1/2} \cosh \Gamma x \cosh \Gamma(L - \xi)}{\sinh \Gamma L}, \qquad \xi \geq x \quad (20)$$

and hence $z_{11} = G_{12}(0,0)$, $z_{12} = G_{12}(0,L)$, while z_{21} and z_{22} follow from $G_{12}(x,\xi)\xi \leq x$ (or in this case by symmetry).

The short circuit parameters can be analogously derived with the aid of $G_{21}(x,\xi)$ when W^a and W^b are suitably chosen. However, it should be noted that both the open-circuit and short-circuit matrices may be also derived more directly with the aid of the transition matrix (5) without reference to (4).

IV. CONCLUSIONS

The Green's matrix procedure, when applied to multiconductor transmission lines, facilitates their analysis in a highly systematic and efficient manner. Thus we note, for example, that conceptually the derivation of the analog of (18) corresponding to more than a single transmission line presents no fresh difficulties, while the use of alternative techniques could prove cumbersome.

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Design and Analysis of a Waveguide-Sandwich Microwave Filter

YUSUKE TAJIMA AND YOSHIHIKO SAWAYAMA

***Abstract*—In 1972 Konishi proposed a unique waveguide filter of sandwich-like construction that was attractive because of its simplicity. This short paper presents an analysis of the sandwich filter that has a conductive sheet with finite thickness sandwiched between waveguide shells. An equivalent circuit is derived, design charts are proposed, and Young's technique is applied to the design of an M-band waveguide filter. The experimental results were found to be in good agreement with the theory and analysis techniques developed herein.**

I. INTRODUCTION

Microwave bandpass filter design techniques have been developed by many authors. For instance, Cohn [1] and Riblet [2] have treated coupled-resonator-microwave filters with narrow and moderate bandwidths, while Young [3] has developed a more general technique that holds for both small and wide bandpass microwave filters. However, the structures of coupled resonator filters are usually fairly complicated because many parts are required and the dimensions of each are critical to the filter's performance.

In 1972 Konishi proposed [4] the "microwave filter with mounted planar circuit in a waveguide," and similar structures have also been suggested bv Meier [5]. These circuits are essentially com-

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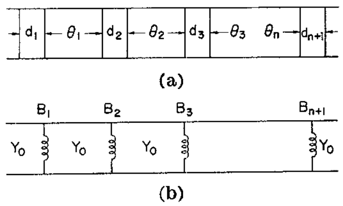

Fig. 1. (a) Configuration of an n-resonator filter in sandwich waveguide structure. (b) Reactance-coupled half-wave filter with shunt-susceptance coupling.

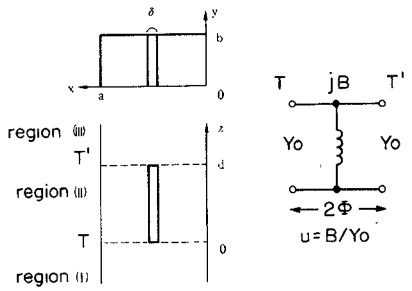

Fig. 2. Inductive strip in a waveguide and its equivalent circuit.

posed of a conductive sheet which is properly designed and sandwiched between the waveguide shells. As the preciseness of dimensions is only required for the conductive sheet, this simplicity is felt to be highly suitable for mass production. Some of the analyses have been shown by the authors [6] and Konishi [7] treated the case of the conductive sheet with infinitesimally small thickness.

This short paper presents an analysis of a sandwich filter which is composed of a sheet of finite thickness with rectangular holes which have the same height as the waveguide. A side view of the filter is illustrated in Fig. 1. Coupling irises in conventional waveguide filters are replaced by a series of inductive strips which are the part of a conductive sheet. In this analysis, the equivalent circuit of the inductive strips is derived. Then Young's technique is applied to a filter design using the results of the analysis. The simulation of the designed filter showed good agreement with the experiments.

II. ANALYSIS

The geometry of the inductive strip within the waveguide is illustrated in Fig. 2. The strip has length d and thickness δ.

When we consider the symmetrization of the structure, the field of our interest can be limited to, in regions (i) and (iii),

$$H_{zm} = \cos\frac{m\pi}{a}x \qquad H_{xm} = \frac{\gamma_m}{\beta_{cm}}\sin\frac{m\pi}{a}x$$

$$E_{ym} = -\frac{j\omega\mu}{\beta_{cm}}\sin\frac{m\pi}{a}x \qquad E_{xm} = H_{ym} = 0, \qquad m = 1,3,5,\cdots, \quad (1)$$

and in region (ii),

$$H_{zn} = \cos\frac{n\pi}{a-\delta}x \qquad H_{xn} = \frac{\gamma_n}{\beta_{cn}}\sin\frac{n\pi}{a-\delta}x$$

$$E_{yn} = -\frac{j\omega\mu}{\beta_{cn}}\sin\frac{n\pi}{a-\delta}x \qquad E_{xn} = H_{yn} = 0, \qquad n = 2,4,6\cdots. \quad (2)$$

We now assume that the conductive strip is thin enough that surface currents at $z = 0$ and $z = d$ flow uniformly in the y direction at $(a-\delta)/2 \le x \le a/2$. We define the surface current density as i_{y1} and i_{y3} and note that

$$H_{1x} = -i_{y1} = \text{const at } z = 0 \quad (3)$$

$$H_{3x} = i_{y3} = \text{const at } z = d. \quad (4)$$

This uniform current assumption, together with the continuity equations of the field at $z = 0$ and $z = d$, results in simultaneous equations:

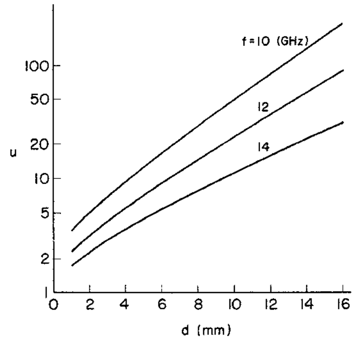

Fig. 3. Susceptance (u) versus strip length (d).

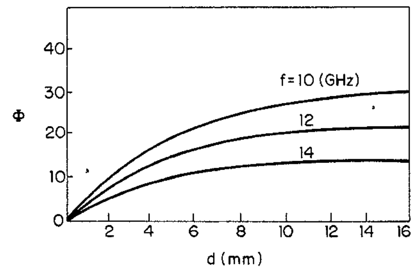

Fig. 4. Φ versus d.

$$\frac{\pi}{2}A_{11} = \sum_n (C_{1n}A_{n2} + D_{1m}B_{n2}) + I_1 i_{y1} \quad (5)$$

$$0 = \sum_n (C_{mn}A_{n2} + D_{mn}B_{n2}) + I_m i_{y1}, \qquad (m = 3,5\cdots) \quad (6)$$

$$0 = \sum_n (D_{mn}A_{n2} + C_{mn}B_{n2}) + I_m i_{y3}, \qquad (m = 1,3,5\cdots) \quad (7)$$

where C_{mn}, D_{mn}, and I_m are known coefficients that can be calculated from dimensions of the strip, while A_{n2}, B_{n2}, i_{y1}, and i_{y3} are unknowns to be solved.

Incident, reflecting, and transmitting waves from region (i) to region (iii) are in a propagating mode (TE_{10}) of amplitudes A_{11}, R_1A_{11}, and $\exp(-\gamma_1 d)A_{13}$, respectively, which can be expressed by the solutions of the simultaneous equations, in the form of

$$(1 + R_1)A_{11} = K\sum_n P_n[A_{n2} + \exp(-\gamma_n d)B_{n2}] \quad (8)$$

$$\exp(-\gamma_1 d)A_{13} = K\sum_n P_n[\exp(-\gamma_n d)B_{n2} + A_{n2}] \quad (9)$$

where K and P_n are constants given by the dimensions of the structure.

From (8) and (9), the S matrix of the strip in a waveguide can be calculated.

III. NUMERICAL RESULTS

Calculations here are of the M-band waveguide (19.05 × 9.525 mm) with a conductive sheet of 0.5-mm thickness, where n is taken as 40.

There are many ways to express the model in equivalent circuits. Here, we took a shunt susceptance circuit with certain electrical-line lengths added to both ports as in Fig. 2, so that a series of strips can be easily designed in the form of the reactance coupled half-wave filter shown in Fig. 1(b). Characteristic impedance of the line is taken equal to the voltage impedance of the waveguide.

Figs. 3 and 4 illustrate u ($= B/Y_0$) and Φ as a function of strip length d for some frequencies. From these figures, a conductive strip of length d is converted to a shunt susceptance circuit with electrical-line length Φ at both sides.

IV. FILTER DESIGN AND EXPERIMENT

A filter is constructed from a series of conductive strips of lengths $d_1, d_2, d_3 \cdots d_{n+1}$, separated from each other by electrical lengths $\theta_1, \theta_2, \cdots \theta_n$, as shown in Fig. 1(a). Using Figs. 3 and 4, this circuit is

TABLE I
DESIGN EXAMPLE

Band edge $f_1 = 11.9$ $f_2 = 12.1$ (GHz)	
VSWR at center frequency 1.5, $g = 0.57$	
Waveguide 19.05×9.525 (mm)	
Sheet thickness 0.5 (mm)	
$d_1 = d_3 = 2.976$	$d_2 = 8.554$
$L_{12} = 12.273$ (mm)	

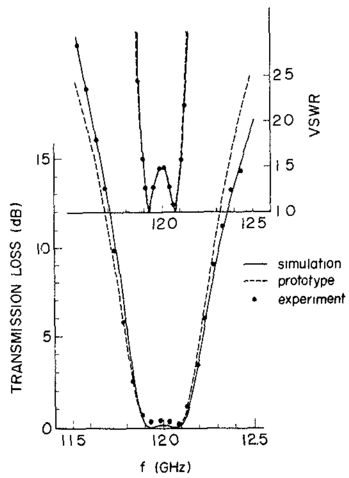

Fig. 5. Designed filter characteristics.

converted to the equivalent circuit shown in Fig. 1(b). Filter design can be deduced from the design technique described in a "synchronously tuned reactance coupled filter" [3]. Applying the technique, step reactances, $u_i = B_i/Y_0$ $(i = 1,n+1)$, are determined from the desired filter characteristics (Chebyshev, maximally flat). The strip length d_i that has a necessary value of u_i can be found from Fig. 3. Separation between the strips θ_i is given by

$$\theta_i = \psi_i - \Phi_i + \psi_{i+1} - \Phi_{i+1} + \frac{\pi}{2} \tag{10}$$

where

$$\psi_i = \tfrac{1}{2} \tan^{-1}\left(\frac{u_i}{2}\right).$$

Here, Φ_i for necessary d_i is given by Fig. 4.

θ_i is frequency dependent and its dependency is greater than the quarter-wave prototype transformer by a factor of

$$\frac{1}{\beta_i} = \frac{2}{\pi}\left[\frac{d\theta_i}{d(\lambda g_0/\lambda g)}\right]_{\lambda g = \lambda g_0} = g_i + g_{i+1} + 1 \tag{11}$$

$$g_i \equiv \frac{2}{\pi}\left[\psi_i - \Phi_i - \frac{d(\psi_i - \Phi_i)}{d(\lambda g_0/\lambda g)}\right].$$

Consequently, the bandwidth of the filter w will be smaller, compared to that of a quarter-wave prototype transformer w_q, and

$$w = \beta w_q. \tag{12}$$

This contraction of the bandwidth must be considered when calculating the prototype bandwidth w_q from desired characteristics. Contraction factor β in (12) should be the smallest among β_i $(i = 1 \cdots n)$ in (11). A value of g is calculated as about 0.57 at 12 GHz giving $\beta = 0.467$.

Table I and Fig. 5 show an example of a filter design. The dotted line in Fig. 5 shows the characteristics of a stepped half-wave filter prototype determined by the input data in Table I, while the solid line shows a simulation of the designed filter. Both of them have almost the same characteristics at the passband, though their skirt cutoff responses differ slightly from each other. Simulation indicates less attenuation at the upper band edge and more attenuation at the lower band edge. This is mainly due to the frequency dependence of u, as is shown in Fig. 3.

The filter was made with the dimensions in Table I. The dimensional accuracy was necessary only for the center conductive sheet with holes. We made it by the arc discharge machinery, but alternately this could be done by the die for the mass production.

In Fig. 5, experimental data are also illustrated by dots, which show good agreement with the simulation. This indicates that the constant current assumption we used during the analysis was good enough for practical use.

V. CONCLUSION

The waveguide-sandwich filter originally proposed by Konishi has an advantage in its structural simplicity. The inductive strips with finite thickness, which are the basic elements of the filter, were analyzed to establish an equivalent circuit. Charts for the filter design were proposed, and a filter was designed for an M-band waveguide. The experiment was in good agreement with the simulation of the designed filter.

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Conduction and Radiation Losses in Microstrip

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***Abstract*—Losses in microstrip on fused-silica and alumina substrates have been experimentally evaluated for various values of stripwidth. Radiation losses in circuits, commonly used for measuring losses in microstrips, depend on the dimensions of the circuit and affect the total losses considerably. Radiation and conduction losses of open-ended line resonators have been separately determined. These measurements have also drawn attention to discrepancies between published theories and our experiments. The effect of surface roughness upon conduction losses has been measured.**

Many measurements of attenuation of microstrip on alumina substrates have been reported previously. In this short paper data will also be presented for microstrip on fused silica. The relevant properties of the fused-silica and alumina substrates are summarized in Table I. The alumina substrates are selected to meet these specifications and are, in most cases, anisotropic with the highest dielectric constant perpendicular to the surface [1]. This latter value is given. The surface roughness of the silica substrates can be easily controlled, providing the possibility to investigate the relation with conduction losses. The conductors consist of two layers. A nickel–phosphorous adhesion layer is deposited by an electroless plating

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